ON $\mathcal{A}$-FOCAL ENTROPY POINTS.

RYSZARD J. PAWLAK

In my talk I will consider points focusing entropy and such that this fact is influenced exclusively by the behaviour of the function around these points (i.e. it is independent from the form of the function at any distance from these points).

Let $X = I^m$ ($I = [0,1]$ and $m = 1, 2, \ldots$) and let $\mathcal{A}$ be the family of all arcs in X. By $\vartheta_{\mathcal{A}}^{Y}$ we will denote the family of all finite sequences of pairwise disjoint arcs contained in $Y \subset X$. For simplicity of notation, let $\vartheta_{\mathcal{A}}$ stand for $\vartheta_{\mathcal{A}}^{X}$. Moreover, $\mathcal{A}|Y = \{K \cap Y : K \in \mathcal{A}\}$.

If $F = (A_1, \ldots, A_m) \in \vartheta_{\mathcal{A}}$ and $f : X \to X$ is a function then we define so called structural matrix $\mathcal{M}_f = [a_{ij}]_{i,j=1}^{m}$ in the following way: $a_{ij} = 1$ if $A_i \underset{f}{\to} A_j$ and $a_{ij} = 0$ otherwise.

A generalized entropy of a function f (not necessarily continuous) with respect to the sequence $F \in \vartheta_{\mathcal{A}}$ is the number $H_f(F) = \log \sigma(\mathcal{M}_f)$ if $\sigma(\mathcal{M}_f) > 0$ and $H_f(F) = 0$ if $\sigma(\mathcal{M}_f) = 0$, where $\sigma(\mathcal{M}_f) = \limsup_{n\to\infty} \sqrt[n]{\operatorname{tr}(\mathcal{M}_f^n)}$.

Let $Y \subset X$ be a nonempty open set. An entropy of f on Y with respect to the family $\mathcal{A}$ is the number

$$H_f(Y) = \sup\left\{\frac{1}{n} H_{f^n}(F) : F \in \vartheta_{\mathcal{A}}^{Y}\right\}.$$

Now, let us introduce the following notation

$$d(f,Y) = \begin{cases} \frac{H_f(Y)}{h(f)} & \text{if } h(f) \in (0,\infty), \\ 1 & \text{if } H_f(Y) = \infty \text{ or } h(f) = 0, \\ 0 & \text{if } H_f(Y) \in [0,\infty) \text{ and } h(f) = \infty. \end{cases}$$

A density of entropy of f at a point x_0 is the number

$$E_f(x_0) = \inf\{d(f,V) : V \in O(x_0)\}.$$

We say that $x_0 \in X$ is an $\mathcal{A}$-*focal entropy point of* f (or briefly: focal entropy point) if $E_f(x_0) = 1$.

The first results will be connected with the fact that each continuous function mapping the unit interval into itself has such kind of points. Moreover, we will discuss the basic properties of the set of all focal entropy points and the possibility of improving functions $f\colon I^m \to I^m$ so that any fixed point of the function becomes its focal entropy point.

Łódź University, Faculty of Mathematics and Computer Science, Banacha 22, 90-238 Łódź, POLAND,

E-mail address: rpawlak@math.uni.lodz.pl