

FRAME BUNDLE APPROACH TO GENERALIZED MINIMAL SUBMANIFOLDS

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ABSTRACT

We extend the notion of r -minimality of a submanifold in arbitrary codimension to u -minimality for a multi-index $u \in \mathbb{N}^q$, where q is the codimension. This approach is based on the analysis on the frame bundle of orthonormal frames of the normal bundle to a submanifold and vector bundles associated with this bundle. The notion of u -minimality comes from the variation of σ_u -symmetric function obtained from the family of shape operators corresponding to all possible bases of the normal bundle. We obtain the variation field, which gives alternative definition of u -minimality. Finally, we give some examples of u -minimal submanifolds for some choices of u and state some relations between generalized symmetric functions σ_u .

More precisely, let M be a Riemannian manifold and L a codimension q submanifold. For a fixed orthonormal basis $e = (e_1, \dots, e_q)$ in the normal bundle to L in M let $\mathbf{A}(e) = (A_1, \dots, A_q)$ be family of shape operators, $A_\alpha = A^{e_\alpha}$. We may associate with the family $\mathbf{A}(e)$ symmetric functions σ_u , where $u \in \mathbb{N}^q$, as follows

$$\det(I + t_1 A_1 + \dots, t_q A_q) = \sum_u \sigma_u t_1^{u_1} \dots t_q^{u_q}.$$

Here σ_u depends on the choice of the basis e . Integrating over all such bases (with respect to the natural measure on the orthonormal frame bundle $O(T^\perp L)$) we get, so called, generalized symmetric functions $\hat{\sigma}_u : L \rightarrow \mathbb{R}$. Critical points of the variation of $\hat{\sigma}_u$ are called u -minimal submanifolds.

We derive the formula for the variation field, which gives alternative definition of u -minimality. For $u = (0, \dots, 0)$ we get the notion of classical minimality. We show existence of u -minimal submanifolds for $u = (0, \dots, 0, 2, 0, \dots, 0)$.

The talk is based on the article [2] and we heavily rely on the results concerning generalized Newton transformation, obtained in the article [1].

REFERENCES

- [1] K. Andrzejewski, W. Kozłowski, K. Niedziałowski, Generalized Newton transformation and its applications to extrinsic geometry, *Asian J. Math.* 20 (2016), No. 2, 293–322.
- [2] K. Niedziałowski, Frame bundle approach to generalized minimal submanifolds, arXiv, <http://arxiv.org/abs/1601.02248>

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