Parameter rigidity of the action of AN on $\Gamma\backslash G$ for higher rank semisimple Lie groups

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Let $M \overset{\rho_0}{\curvearrowleft} S$ be a C^∞ locally free right action of a connected simply connected solvable Lie group S on a closed C^∞ manifold M. Recall that ρ_0 is said to be locally free if the isotropy subgroup of any point of M is a discrete subgroup of S. The set $\mathcal{F}$ of all the orbits of ρ_0 is a C^∞ foliation of M, which is called the orbit foliation of ρ_0. We say ρ_0 is *parameter rigid* if any C^∞ locally free action $M \overset{\rho}{\curvearrowleft} S$ whose orbit foliation coincides with $\mathcal{F}$ is *parameter equivalent* to ρ_0, that is, there exist an automorphism Φ of S and a diffeomorphism F of M such that $F(\rho_0(x, s)) = \rho(F(x), \Phi(s))$ for all $x \in M$ and $s \in S$ and F preserves each leaf of $\mathcal{F}$ and is C^0 homotopic to the identity map of M through C^∞ maps preserving each leaf.

For example a linear flow on a torus is parameter rigid if and only if the velocity vector of the flow at a point satisfies the Diophantine condition.

In two papers published in 1994, A. Katok and R. J. Spatzier proved the following theorem.

Theorem 1. *Let G be a connected semisimple Lie group with finite center of real rank at least 2 without compact factors or simple factors locally isomorphic to $\mathrm{SO}_0(n, 1)$ $(n \geq 2)$ or $\mathrm{SU}(n, 1)$ $(n \geq 2)$ and Γ be an irreducible cocompact lattice in G. Let $G = KAN$ be an Iwasawa decomposition. Then the action $\Gamma\backslash G \curvearrowleft A$ by right multiplication is parameter rigid.*

Recently I proved the following.

Theorem 2. *Let G be a connected semisimple Lie group with finite center of real rank at least 2 without compact factors or simple factors locally isomorphic to $\mathrm{SO}_0(n, 1)$ $(n \geq 2)$ or $\mathrm{SU}(n, 1)$ $(n \geq 2)$ and Γ be an irreducible cocompact lattice in G. Let $G = KAN$ be an Iwasawa decomposition. Then the action $\Gamma\backslash G \curvearrowleft AN$ by right multiplication is parameter rigid.*

The major difference in the proof comes from the noncommutativity of AN. I will explain how to prove Theorem 2 in the talk.

The proof is basically a combination of a sufficient condition for parameter rigidity of Maruhashi, cohomology vanishing results of Katok–Spatzier and Kanai and rigidity theorems of quasiisometries of symmetric spaces of Pansu, Kleiner–Leeb, Farb–Mosher and Reiter Ahlin.