A TRACE FORMULA FOR CODIMENSION ONE FOLIATIONS WITH SIMPLE FOLIATED FLOWS

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Let $\mathcal{F}$ be a smooth, transversely oriented, codimension one foliation on a compact smooth manifold M and ϕ a foliated flow on $(M, \mathcal{F})$. Denote by $\mathrm{Fix}(\phi)$ the fixed point set of ϕ. Let M^0 be the $\mathcal{F}$-saturation of $\mathrm{Fix}(\phi)$, and $M^1 = M \setminus M^0$. We will assume that ϕ is simple, which means that all of its fixed points and closed orbits are simple, and its orbits in M^1 are transverse to the leaves. In this case, M^0 is a finite union of compact leaves, and M^1 has finitely many connected components, denoted by M_l. One can construct a bundle-like metric g^1 on M^1 such that each M_l with respect to this metric is a manifold of bounded geometry, and the restriction $\mathcal{F}_l$ of $\mathcal{F}$ to M_l is a foliation of bounded geometry. In addition, without loss of generality, we can assume that g^1 has a particular form in a neighborhood of M^0.

Denote by $d_{\mathcal{F}_l}$ and $\delta_{\mathcal{F}_l}$, respectively, the leafwise derivative and the leafwise coderivative, acting in $C^\infty(\bigwedge T\mathcal{F}_l{}^*)$, and set $D_{\mathcal{F}_l} = d_{\mathcal{F}_l} + \delta_{\mathcal{F}_l}$. Let $\mathcal{A}$ be the Fréchet algebra of functions $\psi : \mathbb{R} \to \mathbb{C}$ that can be extended to entire functions on $\mathbb{C}$ such that, for each compact subset K of $\mathbb{R}$, the set $\{\, x \mapsto \psi(x + iy) \mid y \in K \,\}$ is bounded in the Schwartz space $\mathcal{S}(\mathbb{R})$. For any $\psi \in \mathcal{A}$ and $f \in C^\infty_{\mathrm{c}}(\mathbb{R})$, consider the operator $P : C^\infty_c(M^1; \bigwedge T\mathcal{F}^{1*}) \to C^\infty(M^1; \bigwedge T\mathcal{F}^{1*})$, whose restriction to $C^\infty_c(M_l; \bigwedge T\mathcal{F}_l^*)$ is given by

$$P_l = \int_{-\infty}^{\infty} \phi^{t*} \psi(D_{\mathcal{F}_l}) f(t)\, dt.$$

One can show that the Schwartz kernel of P extends to a smooth function on $M \times M^1 \cap M^1 \times M$ and has singularity at $M^0 \times M^0$. In particular, the operator P is not of trace class in $L^2(M; \bigwedge T\mathcal{F}^*)$.

To define a trace of P, we use the machinery of pseudodifferential b-calculus on manifolds with boundary developed by R. Melrose in his book on the Atiyah-Patodi-Singer theorem. For each l, M_l is the interior of a connected compact manifold M_l^{c} with boundary and the foliation $\mathcal{F}_l$ extends to a smooth foliation $\mathcal{F}_l^{\mathrm{c}}$ on M_l^{c} tangent to the boundary. We prove that each P_l defines an operator of the class $\Psi_{\mathrm{b}}^{-\infty}(M_l^{\mathrm{c}}; \bigwedge T\mathcal{F}_l^{\mathrm{c}*})$ of b-pseudodifferential operators of order $-\infty$. R. Melrose constructed an extension ${}^{\mathrm{b}}\mathrm{Tr}$ of the trace functional to $\Psi_{\mathrm{b}}^{-\infty}(M_l^{\mathrm{c}}; \bigwedge T\mathcal{F}_l^{\mathrm{c}*})$, called *b*-trace. These facts allow us to introduce the Lefschetz distribution of ϕ and study the associated trace formula. In my talk, I will report on the recent progress in this direction.

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