APPROXIMATING C^0-FOLIATIONS BY CONTACT STRUCTURES

There is fundamental relationship between foliation theory and contact topology that was discovered by Eliashberg and Thurston in the late 90's: they showed that any cooriented foliation $\mathcal{F}$ (of class C^2) on an orientable, closed 3-manifold can be approximated by contact structures. More precisely, the tangent distribution $T\mathcal{F}$ can be approximated by contact structures in the C^0-sense. Amongst other things this played a central role in Mrówka and Kronheimer's proof of the Property P conjecture, which shows that surgery on a knot K yields a homotopy sphere if and only if K is the unknot, the surgery coefficient is ± 1 and the trace of the surgery is S^3.

The power of their theory comes from the following corollary to their approximation theorem (which does not require any regularity of the foliation):

Corollary 1. *Let $\mathcal{F}$ be a taut coorientable foliation on a closed orientable 3-manifiold. Then any sufficiently close contact structure is (universally) tight.*

The regularity assumption in Eliashberg-Thurston's result is *a priori* quite strong and many constructions (surgery, blow-up, gluing...) yield foliations that are only of class C^0, in the sense that the leaves are smooth, but the tangent distribution is only continuous. For example on rational homology spheres that are not graph manifolds it is not known in general that any taut foliations of class C^2-exist, in the case that there are taut C^0-foliations. Moreover, one cannot approximate C^0-foliations by ones of class C^2 in general, with obstructions coming from Kopell's Lemma for example.

Eliashberg and Thurston already noted in their book on Confoliations that one can approximate foliations that are smooth away from a finite collection of compact leaves and they also write

> "However it is feasible that the result holds without any assumptions about the smoothness of the foliation."

In this talk we will discuss the following generalisation of Eliashberg-Thurston result to C^0-foliations, which confirms Eliashberg and Thurston's comment above and was also independently proven by Kazez-Roberts:

Theorem 2 (Bowden, Kazez-Roberts)**.** *Let $\mathcal{F}$ be a C^0-foliation on a closed 3-manifold that is neither the foliation by spheres on $S^2 \times S^1$ nor a non-minimal foliation by planes on T^3. Then $T\mathcal{F}$ can be C^0-approximated by positive and negative contact structures.*