

FOLIATIONS, FRACTALS AND DYNAMICS

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ABSTRACT.

One of the most fundamental invariants of the dynamics of a continuous map $f : X \rightarrow X$ is its topological entropy which measures the complexity of f . When the entropy is positive, it reflects some chaotic behavior of the map f . There exists a corresponding notion of the topological entropy for a group or pseudogroup action. For any foliated space (M, F) , the action of the holonomy pseudogroup H on the complete transversal T contains complete information about the dynamics of (M, F) .

Ghys, Langevin and Walczak in the celebrated paper [4] proved that a foliation can be considered as a generalized dynamical system. A codimension one foliation with positive geometric entropy admits a resilient leaf with very complicated geometry and its exceptional minimal set is a Cantor set.

The theory of foliations of codimension greater than one starts with the paper by Thurston [5], there are many particular foliations in higher codimension studied by Molino, Epstein, Blumenthal and others. However for foliations of codimension greater than one there are very few results on its dynamics (see [1], [2],[3]) and much more open problems. From the dynamical point of view there are many differences between codimension one and higher codimension foliations.

Most fractals can be realized as minimal sets of codimension greater than one foliated spaces. The geometry of leaves, fractals being transversals and dynamics are interrelated. In the talk I will present old and new results related with the dynamics of codimension greater than one foliated spaces.

REFERENCES

- [1] A. Biś, H. Nakayama, P. Walczak, *Modelling minimal foliated spaces with positive entropy*, Hokkaido Math. J. 36 (2007), 283-310.
- [2] A. Biś, S. Hurder and J. Shive, *Generalized Hirsch foliations*, in *Foliations 2005* ed. by P. Walczak et al., World Scientific 2006, 71-108.
- [3] A. Biś, *Dynamics of Foliated Spaces in Codimension Greater than One*, Fields Institute Communications, Volume 51, 2007, 249-268.

- [4] E. Ghys, R. Langevin and P. Walczak, *Entropie géométrique des feuilletages*, Acta Math. 160 (1988), 105-311.
- [5] W. Thurston, *The theory of foliations of codimension greater than one*, Comment. Math. Helv. 49 (1974), 214-231.