A Chern–Weil construction for derivatives of characteristic classes

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Secondary characteristic classes for foliations are usually constructed by using Bott connections via the Chern–Weil (and the Chern–Simons) construction. Given an infinitesimal deformation of a foliation, we can define the derivatives of characteristic classes with respect to it [6], [7]. They are defined by means of differential forms, however, the construction involves combinatorial arguments and seemingly different from that of their primitives, namely, usual characteristic classes [7], [2]. In this talk, I will introduce a certain vector bundle which is an analog of 2-tangent bundles TTM for manifolds. Once an infinitesimal deformation is given, one can define a connection on the bundle with which a characteristic homomorphism can be constructed in the usual way. The homomorphism gives not only derivatives but some exotic classes such as the Fuks–Lodder–Kotschick class '$\dot{h}_1 h_1 c_1^q$' [5], [9], [8]. If we deal with the Godbillon–Vey class or the Bott class, the derivatives are known to be represented by projective Schwarzians [10], [1]. This is also explained in a similar framework as above [3]. I will discuss it if the time allows.

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